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CALCULATION OF THE TEMPERATURE FIELDS IN A BLADE OF A HIGH-TEMPERATURE GAS TURBINE DURING INTERNAL COOLING

L. M. Zysina-Molozhen and M. P. Polyak

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SYMBOLS USED

t_g-gas temperature (surrounding medium), °C ($t_g = t_r$)

t_s-temperature of surface of blade at given point, °C

t_c-temperature of cooling medium, °C

α-heat transfer coefficient from gas to external blade surface, kcal/m^2 °C·hr

α'-heat transfer coefficient from internal surface of blade to cooling medium, kcal/m^2·hr °C

λ-coefficient of heat conductivity of blade material, kcal/m^2·hr °C

n-external normal to blade contour

$\frac{\partial t_s}{\partial n}$-value of temperature gradient at blade contour

dF-element of surface through which elementary quantity of heat dQ is transmitted

t_{iS}-temperature of i-th point on contour

r_i-distance from i-th point on contour to a certain point M

$\cos(r_i, n)$-cosine of angle between ray from point M to i-th point of contour, and external normal to contour at this point

S-length of contour

S_0-external contour of blade

$S_1, \ldots, S_6$-internal contours of six cooling channels

$\alpha(x,y)$-value of heat transfer coefficient from gas to blade, variable over contour

$\alpha_1, \ldots, \alpha_6$-coefficients of heat transfer from walls of internal channels to cooling medium

Q-total quantity of heat applied to blade

q-density of heat flux

I-electric current

i-density of electric current

u-electrical potential

$\sigma = 1/\rho$-coefficient of electrical conductivity of material of model blade (ρ is the specific resistance of the electrically conductive paper)

L-linear dimension

λ-coefficient of thermal conductivity of original blade

α-coefficient of heat transfer

R_e-electrical resistance

$F_e = l_i\delta$-local electrical model cross-section

δ = const-thickness of electrically conductive paper

l_i-length of sector of contour of model blade cross-section

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CALCULATION OF THE TEMPERATURE FIELDS IN A BLADE OF A HIGH-TEMPERATURE GAS TURBINE DURING INTERNAL COOLING /261[1]

L. M. Zysina-Molozhen and M. P. Polyak
Central Boiler-Turbine Institute imeni I.I.Polzunov

ABSTRACT. Calculation of the temperature fields in a cross-section of a gas turbine blade with an internally flowing coolant. The problem consists of calculating the temperature fields within the body of a blade of arbitrary shape and with an arbitrary number of internal cooling channels during variation of the local heat-transfer coefficients along the blade contour. The results make possible an evaluation of the effectiveness of the cooling system in terms of the optimum distribution of the cooling channels. The solution is based on determining the temperature function which corresponds to a Laplace equation with certain boundary conditions. The calculation was performed on a digital computer, and the method of programming is described in detail.

One of the most heavily loaded elements of a gas turbine is the moving blade. Therefore, the problem of provision of effective cooling of moving blades in gas turbines is one of the most important problems to be solved in the production of high temperature gas turbines.

One of the most promising systems for cooling blades is the system of internal cooling using an intermediate heat transfer agent moving through internal cooling channels in the blades. From the mathematical point of view, the problem of the temperature field in such a blade is a complex spatial problem of stationary heat transfer with variable boundary conditions along the contour and height of the blade. This problem has not yet been theoretically solved due to the great mathematical difficulties involved. The solution of a simpler problem, that of determining the temperature field in cross-sections of cooled blade profiles, is also of great interest for turbine construction.

The solution of this problem for cylindrical blades allows us to determine the temperature field throughout the entire fin of the blade; for curved blades, using the method of cylindrical cross-sections, we can use this solution to determine the temperature field in various cross-sections through

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[1] Numbers in the margin indicate pagination in the foreign text.

the height of the blade.

In blades with internal cooling, the temperature gradients in the cross-section of the profile may reach several hundred degrees, and this phenomenon cannot be ignored. In order to increase the reliability of the operation of such blades, it is necessary that a reduction in the temperature gradients in the cross-section of the profile be produced by proper selection of the location, number, form and size of cooling channels.

In experimental investigations of heat transfer of turbine blades, the average coefficients of heat transfer have been primarily studied. these coefficients can be used, for example, to estimate heat transfer to the turbine rotor. In planning a blade cooling system, however, it is extremely important /262 to have the ability to calculate heat transfer and temperature fields within the body of the cooled blade in consideration of the inconstant nature of local heat transfer through the lateral surface, since the nature of the flow of the gas in the interblade channel of the turbine results in quite essential differences in the intensities of heat transfer through various sectors of the surface past which the gas stream flows. A cooling system planned without knowledge of the distribution of temperatures through the body of a blade may increase the unevenness of temperature within the blade and, consequently, reduce its mechanical strength.

At the present time it has become quite possible to calculate the thermal effectiveness of any network of profiles on the basis of calculations of conditions in the boundary layer developing along the contour of the profiles around which the fluid flows. The method of determining the coefficients of heat transfer based on calculation of the boundary layer [1] takes better consideration of the physical aspect of the process. Since this method has been programmed for the high speed "Ural-1" electronic computer, the performance of a series of calculations of local and average values of heat transfer coefficients for various networks of profiles represents no particular difficulty, being an operation requiring only a few minutes.

The problem of determination of the temperature field within a hollow turbine blade of arbitrary form, around which a gas flows, where the third order boundary conditions are fixed for the external and internal contours, has been solved by O. I. Golub'yeva [2] on the assumption that the heat transfer coefficients from the gas to the external surface are constant throughout the contour. We used the method developed in [2] to solve the more complex problem of the temperature field within a blade of arbitrary form with arbitrary number of internal cooling channels of any form considering variability of local heat transfer coefficients over the contour. The practical solution of this problem was possible only by programming the method for the high speed electronic computer. In the solution of the problem, the end surfaces of the blade were considered to be insulated.

As a result of this assumption, the temperature values produced are somewhat elevated, since we did not consider heat transfer to the wheel rim by conduction. However, the cooling effect of the rim of the wheel is noticeable only for a slight distance from the rim, particularly for heat resistant steels, which have low heat conductivity. Also, the inaccuracy produced as a result of this assumption increases the strength reserve.

The solution of the problem consists of determination of function t (temperature) satisfying the Laplace equation /263

$$\Delta t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 \qquad (1)$$

and the following boundary conditions:

1. The elementary quantity of heat supplied by convection from the surrounding medium to an element of the exterior surface of the blade is equal to the elementary quantity of heat conducted away from the blade by heat conduction

$$dQ = \alpha (t_r - t_S)\, dF = \lambda \frac{\partial t_S}{\partial n} dF. \qquad (2)$$

2. The elementary quantity of heat transmitted by heat conductivity to an element of the internal surface of the blade (wall of the internal channel) is equal to the elementary quantity of heat conducted away from it by convection of the cooling medium

$$dQ = \alpha' (t_S - t_C)\, dF = -\lambda \frac{\partial t_S}{dn} dF. \qquad (3)$$

In this case, one of the two main boundary problems for the Laplace equation obtains; the Neuman problem, which is stated as follows: find function t, harmonic in area D, for which the normal derivative $\partial t/\partial n$, i.e. the derivative with respect to the direction of the normal to the contour, has fixed values over contour S:

$$\left(\frac{\partial t}{\partial n}\right)_S = f_1 (t_S). \qquad (4)$$

In this case, the Laplace equation (1) can be reduced to an integral equation

$$t_{iS} = \frac{1}{2\pi} \left[\int_S \frac{\cos (r_i, n)}{r_i} dS - \int_S \ln r_i \frac{\partial t_{Si}}{\partial n} dS \right]. \qquad (5)$$

This equation is produced as a result of a limit transfer from the corresponding equation for temperature at a certain point within the area bounded by contour S, using the known properties of the harmonic functions.

We have solved the problem of determining the temperature field in the cross-section of a blade with six cooling channels (the number of channels is arbitrary). In this case, the contour through which integration is performed /264
consists of seven parts: the external contour of the blade and six internal contours of cooling channels. Equation (5) for conditions (2) and (3) can be written as follows

$$
\begin{aligned}
2\pi t_{Si} = {} & \int_{S_0} t_S \frac{\cos(r_i n)}{r_i} dS - \int_{S_0} \ln r_i \frac{\alpha(x, y)}{\lambda(x, y)} (t_r - t_S)\, dS + \\
& + \int_{S_1} t_S \frac{\cos(r_i n)}{r_i} dS - \int_{S_1} \ln r_i \frac{\alpha_1}{\lambda(x, y)} (t_{c1} - t_S)\, dS + \\
& + \cdots + \cdots + \int_{S_6} t_S \frac{\cos(r_i n)}{r_i} dS - \int_{S_6} \ln r_i \frac{\alpha_6}{\lambda(x, y)} (t_{c6} - t_S)\, dS.
\end{aligned}
\tag{5'}
$$

Integral equation (5) can be solved using the approximate Fredholm method. This requires that the internal and external contours be divided into n + m sectors (n sectors on the external contour and m on the internal contour), in each of which temperature t_s is assumed constant and equal to the average temperature over the entire sector. Equation (5') is made up for each such sector and, as a result of solution of the system of n + m equations with n + m unknowns, we find the temperatures t_{s1}, t_{s2}, ..., t_{sn}, $t_{s,n+1}$..., $t_{s,n+m}$ over the profile contour. Of course, the accuracy of the solution depends on the number of sectors into which the contour is divided. The integrals for contours $S_0, S_1, \ldots, S_6$ will be calculated as sums of integrals for the n + m sectors. Equation (5') then takes on the form

$$
\begin{aligned}
& \varphi_{i1} t_1 + \varphi_{i2} t_2 + \cdots + (\varphi_{ii} - 2\pi) t_i + \cdots + \varphi_{in} t_n - \\
& - \varphi_{iS_0} t_r - \varphi_{iS_1} t_{c1} - \varphi_{iS_2} t_{c2} - \cdots - \varphi_{iS_6} t_{c6},
\end{aligned}
\tag{6}
$$

where ϕ represents the following coefficients:

$$
\varphi_{i1} = \int_1 \frac{\cos(r_i, n)}{r_i} dS + \int_1 \frac{d}{\lambda} \ln r_i\, dS,
$$

. . . .

$$
\varphi_{in} = \int_n \frac{\cos(r_i, n)}{r_i} dS + \int_n \frac{\alpha}{\lambda} \ln r_i\, dS,
$$

$$\varphi_{i,n+1}=\int\limits_{n+1}\frac{\cos(r_i,n)}{r_i}\,dS+\int\limits_{n+1}\frac{\alpha_1}{\lambda}\ln r_i\,dS,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\varphi_{i,n+m}=\int\limits_{n+m}\frac{\cos(r_i,n)}{r_i}\,dS+\int\limits_{n+m}\frac{\alpha_6}{\lambda}\ln r_i\,dS,$$

$$\varphi_{iS_0}=\int\limits_{1}\frac{\alpha}{\lambda}\ln r_i\,dS+\int\limits_{2}\frac{\alpha}{\lambda}\ln r_i\,dS+\cdots+\int\limits_{n}\frac{\alpha}{\lambda}\ln r_i\,dS,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\varphi_{iS_6}=\int\limits_{n+j}\frac{\alpha_6}{\lambda}\ln r_i\,dS+\cdots+\int\limits_{n+m}\frac{\alpha_6}{\lambda}\ln r_i\,dS.$$

The subscript i means that the temperature is determined for the i-th point on the contour, while the numbers 1,2,...,n,...,(n + m) indicate the sector for which the integrals are calculated.

For each sector where t_s = const, α and λ are also assumed constant, and the calculation of the integrals is greatly simplified.

Thus composing the (n + m) equations (for each sector), we produce a system of linear equations which must be solved in order to determine the functions $t_1, t_2, \ldots, t_n, \ldots, t_{n+m}$ over the contour:

$$\begin{gathered}
(\varphi_{11}-2\pi)\,t_1+\varphi_{12}t_2+\cdots+\varphi_{1n}t_n+\varphi_{1,n+1}t_{n+1}+\cdots+\\
+\varphi_{1,n+m}t_{n+m}-\varphi_{1S_0}t_{\mathrm{r}}-\varphi_{1S_1}t_{c1}-\varphi_{1S_2}t_{c2}-\\
-\cdots-\varphi_{1S_6}t_{c6}=0;\\
\varphi_{21}+(\varphi_{22}-2\pi)\,t_2+\cdots+\varphi_{2n}t_n+\varphi_{2,n+1}t_{n+1}+\cdots+\\
+\varphi_{2,n+m}t_{n+m}-\varphi_{2,S_0}t_{\mathrm{r}}-\varphi_{2,S_1}t_{c1}-\\
-\varphi_{2S_2}t_{c2}-\cdots-\varphi_{2S_6}t_{c6}=0;\\
\cdots\cdots\cdots\cdots\cdots\cdots\\
\varphi_{n1}t_1+\varphi_{n2}t_2+\cdots+(\varphi_{nn}-2\pi)\,t_n+\varphi_{n,n+1}t_{n+1}+\\
\cdots+\varphi_{n,n+m}t_{n+m}-\varphi_{n,S_0}t_{\mathrm{r}}-\varphi_{n,S_1}t_{c1}-\\
-\varphi_{n,S_2}t_{c2}-\cdots-\varphi_{n,S_6}t_{c6}=0;\\
\cdots\cdots\cdots\cdots\cdots\cdots
\end{gathered}$$

$$\varphi_{n+m,1} t_1 + \varphi_{n+m,2} t_2 + \cdots + \varphi_{n+m,n} t_n + \varphi_{n+m,n+1} t_{n+1} + \\ + \cdots + (\varphi_{n+m,n+m} - 2\pi) t_{n+m} - \varphi_{n+m,S_0} t_r - \varphi_{n+m,S} t_{c1} - \\ - \varphi_{n+m,S_2} t_{c2} - \cdots - \varphi_{n+m,S_6} t_{c6} = 0.$$

In order to solve this system we must first determine the component coefficients ϕ.

Using the known temperatures on the contour, we can determine the temperature at any point in the cross-section of the blade:

$$t_l = \frac{1}{2\pi} \Big[\varphi_i t_{S_1} + \varphi_{i2} t_{S_2} + \cdots + \varphi_{i,n+m} t_{S,n+m} - \\ - \varphi_{lS_0} t_r - \varphi_{lS_1} t_{c1} - \cdots - \varphi_{lS_6} t_{c6} \Big]. \quad (7)$$ /266

The complexity and difficulty of this numerical method of solution are obvious, and its practical application, considering the variability of α_s through the contour and the great volume of computational work required, is possible only where high speed electronic computers are available.

The method described above was programmed by us for the "BESM" electronic computer. The program is designed to determine the temperature at 100 points on the contour and any number of internal points of the cross-section. The calculation requires that the following quantities be assigned: coordinates X and Y of the points of division of the contours (external and internal), the values of the coefficients of heat transfer at these points, the values of the coefficient of heat conductivity of the blade material, the gas temperature and the temperature of the coolant.

The machine time required for a determination of temperature values at 200 points in the cross-section is three hours. Punching of the initial numerical material requires 30 minutes.

Placement of the initial data on punched cards allows rapid replacement of individual cards to be performed in case of changes in various input quantities, i.e. calculation of numerous variants of the problem can be performed in succession. The results of the solution are printed by a high speed printer onto strip paper in the form of a column of numbers. For the points on the contour, the temperature values at the 100 points are printed out in order. For the internal points, the coordinates (X,Y) of each point and the corresponding temperature value at the point are printed out in order.

We have solved the problem of determining the temperature field in the body of a blade during intensive cooling by liquid sodium filling six internal channels. The external contour of the cross-section was divided into

40 sectors (20 on the back edge and 20 on the curved side). Analysis of the distribution of α_s over the external contour for various blade profiles showed that the division of the external contour into 40 sectors, over each of which it is assumed that α = const, is quite sufficient for reproduction of the true /267 picture of distribution of α_s. The program for the method of calculating α_s, to be run on the "Ural-1" computer, also calls for the separation of the contour into 40 sectors (the values of α_s are determined at 40 points on the contour according to the predetermined values of velocity at each point). Each of the six cooling channels was divided into ten sectors.

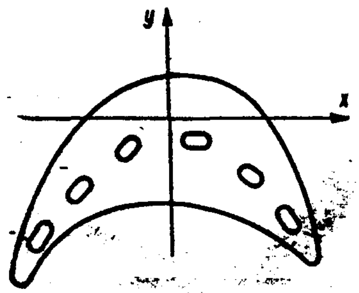

Figure 1. Graph for composition of heat conductivity equation.

Figures 2 and 3 show one of the distributions of velocities and local heat transfer coefficients around the contour of the blade cross-section, produced by calculation for this problem. As we see, there is an essential unevenness in the distribution of α_s. The maximum values are attained at the leading and trailing edges of the blade, the values of α_s at the edges exceeding the values of α_s in the central portion of the blade by a factor of 3 or even 4, indicating that the blade edges must withstand the most difficult conditions as concerns temperatures, and therefore need more intensive cooling. One distribution of the temperatures in the cross-section of the blade produced by calculation using our program on the "BESM" computer for t_g = 1200°C, t_c = 500°C and α from the channel walls to the liquid sodium α_{Na} = 160,000 kcal[1]/m^2·hr°C is shown on figure 4. As we see, the maximum value of temperature, as was to be expected, occurs on the edges, where the most intensive heat exchange between the gas and the wall occurs. At the back edge, /268 the temperature maximum is 899°C; at the front edge the maximum value is 850°C (lower than at the back edge, in spite of the higher value of α_s in this area, due to the proximity of the cooling channel). In the central portion of the blade, the temperature is 550-650°C. The temperature of the cooling channel walls, due to the high heat transfer coefficient from walls to sodium, does not exceed the temperature of the sodium by more than 10°C in the central portion of the profile and 30°C in the end channels.

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[1] This value of α_{Na} was produced by calculation using the Leighill formula: $Nu = 0.0192\ (GrPr)^{0.4}$.

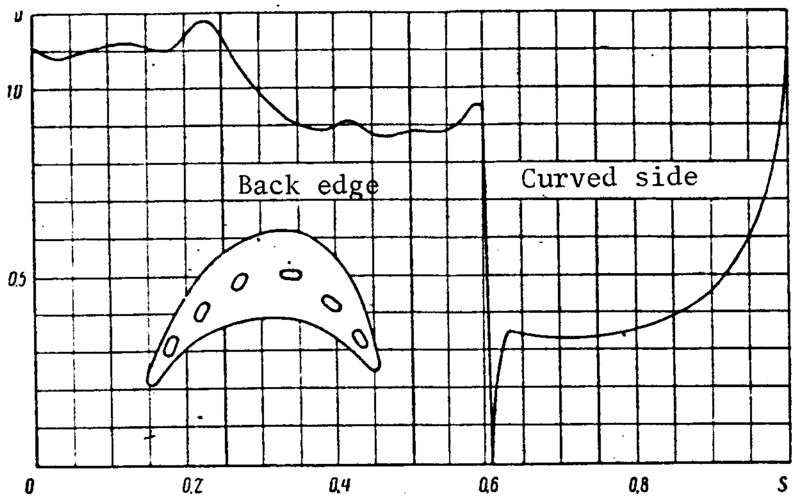

Figure 2. Distribution of Velocity Along Contour of Moving Blade Profile

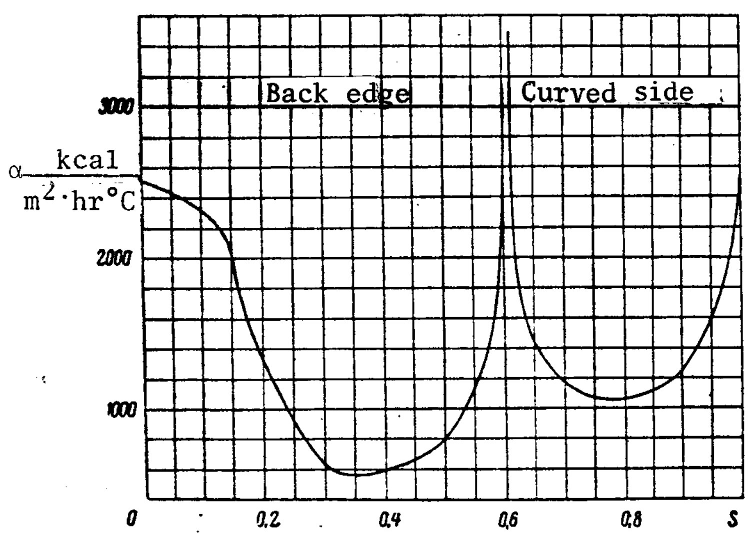

Figure 3. Distribution of Heat Transfer Coefficient Along Contour of Moving Blade of GT-1200

The results of calculations for the concrete conditions indicate possible directions for changes in the location and configuration of cooling channels in order to produce a more even temperature field.

In order to check the accuracy of the method of calculation suggested and to improve the final program, the problem was modeled using a type EGDA-6/53 electrothermal analog device[1].

During the modeling, the condition of similarity between the electrical and thermal problems was maintained, as expressed by the relationship

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[1] The solution of the problem on the electrothermal analog device was performed by L. S. Petukhov. The installation and model are described in detail in work [3].

$$\frac{c_q}{c_T c_\lambda c_z} = 1 \quad \text{and} \quad \frac{c_\lambda}{c_\alpha c_z} = 1. \qquad (8)$$

$$c_q = \frac{Q}{I} = \frac{q}{i};\ c_T = \frac{t}{u};\ c_\lambda = \frac{\lambda}{\sigma};$$ /270

$$c_L = \frac{L_{orig}}{L_{model}};\ c_\alpha = \frac{\alpha}{\frac{1}{R_e F_e}}.$$

The distribution of the local heat transfer coefficient over the contour of the blade was modeled by electrical conductivities $S_i = 1/R_{e_i} F_{e_i}$, where R_{e_i} was calculated according to the formula $R_{e_i} = c_\alpha (1/\alpha_i F_{e_i})$ and was connected up using buses to the corresponding sectors on the contour of the electrical model. In all, 40 such buses were located about the contour of the blade profile.

The desired field of isotherms in the cross-section of the original blade was determined after conversion of the field of distribution of electrical potentials prooduced on the model using the formula $T = c_T u$, where coefficient c_T was determined using relationship (8) according to the coefficients c_q, c_α and c_L, which had been determined in advance.

The results of these calculations for the particular example shown on figure 4 are illustrated on this same figure by the dotted lines. As we see, the data agree well with each other, which indicates that the program is well composed and that the suggested method of calculation is satisfactory. Thus, we can say that this suggested method of calculation, programmed for electronic computer, allows us to perform calculations of temperature fields in a flat cross-section of a blade with any number of internal cooling channels with an accuracy no less than the accuracy of the electrical integrator, but in a considerably shorter period of time, with no need for the preparation of an electrical model. It is also possible to perform rapid computational investigation of a series of variants. This factor allows a large number of variant calculations to be performed, as are necessary in design development of a blade cooling system, in a comparatively short period of time.

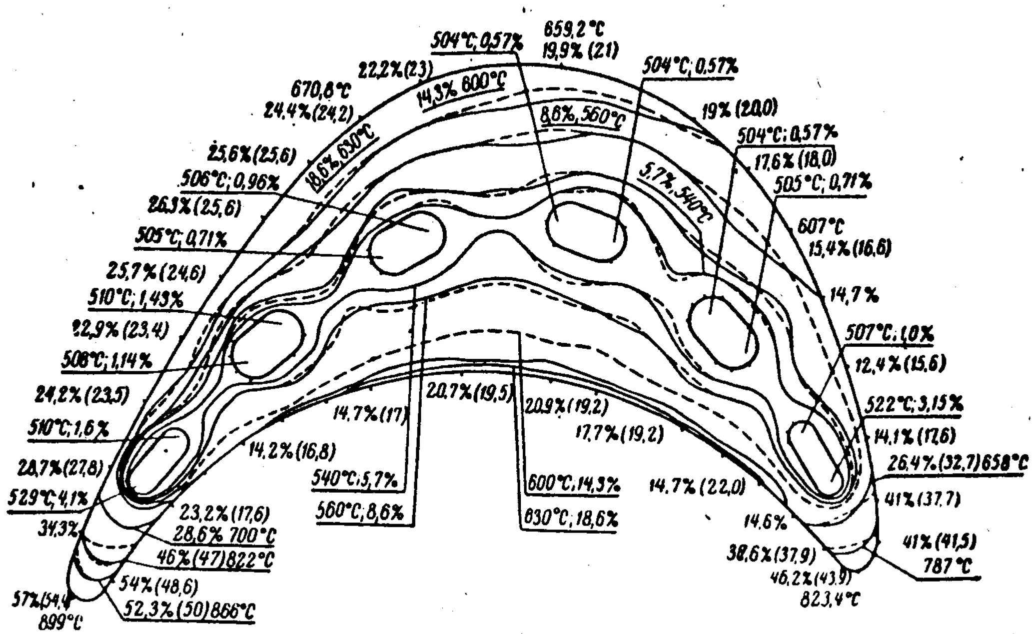

Figure 4. Distribution of Temperature Through Body of blade with Six Cooling Channels

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2. O. I. Golub'yeva, *Trudy TsIAM*, No. 129, 1947.
3. L. S. Petukhov, *Trudy TsKTI, Kotloturbostroyeniye*, No. 47, 1964.

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